35. *Prove that if*

$$\frac{a_0}{1}+\frac{a_1}{2}+\cdots+\frac{a_n}{n+1}=0,$$

then

$$a_0+a_1x+\cdots+a_nx^n=0$$

for some x in $[0,1]$.

Let $f(x)=a_0+a_1x+\cdots+a_nx^n$, and let $g(x)=a_0x+\frac{a_1}{2}x^2+\cdots+\frac{a_n}{n+1}x^n$. Since each term has an x in it, $g(0)=0$. Also, $g(1)=a_0+\frac{a_1}{2}+\cdots+\frac{a_n}{n+1}$. But by our assumption this means $g(1)=0$. Finally we note that since g is a polynomial it is continuous and differentiable on $(0,1)$, and so Rolle's Theorem applies: there exists $x_0\in(0,1)$ such that $g'(x_0)=0$. But $g'(x)=f(x)$, so $f(x_0)=0$.

Do you have any idea how long it took me to stumble upon this?