3. Prove that if X is path-connected and $f : X \to Y$ is a map, then $f(X)$ is path-connected.
Let $y_1, y_2 \in f(X)$.
Then there exists $x_1, x_2 \in X$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$.
Since X is path-connected, there's $\phi : [0,1] \to X$ such that $f(x_1) = x_1$ and $f(x_2) = x_2$.
Consider $f \circ \phi : [0,1] \to Y$.
By Theorem II.3.2, the composition of continuous functions are continuous.
Therefore $f \circ \phi$ is a continuous function from $[0,1]$ to Y, with $f \circ \phi(0) = f(x_1) = y_1$ and $f \circ \phi(1) = f(x_2) = y_2$.
I think we get #2 for free from this one.