2. *Let X be a set and let $\mathcal{T}$ be the family of subsets U of X such that $X \backslash U$ is finite, together with the empty set $\varnothing$. SHow that $\mathcal{T}$ is a topology.*

(a) $X \backslash X = \varnothing$ has a finite number of elelements (namely zero), and $\varnothing \in U$ by definition.

(b) Let $\mathcal{F}$ be the union of a family of subsets of $\mathcal{T}$, and suppose A is a member of $\mathcal{F}$. Then $A \subseteq \mathcal{F}$. But this means that $X \backslash \mathcal{F} \subseteq X \backslash A$, and since $X \backslash A$ is finite so must $X \backslash \mathcal{F}$. Therefore $X \backslash \mathcal{F}$ is finite and $\mathcal{F} \in \mathcal{T}$

(c) Let $A_1, A_2, \ldots, A_n \in \mathcal{T}$. We know that $X \backslash A_1$ is finite because $A_1 \in \mathcal{T}$. Suppose $\bigcap_{k=1}^{n-1} A_k \in \mathcal{T}$. Then$X \backslash \{A_1 \cap A_2 \cap \ldots \cap A_{n-1}\}$ is finite; let's say it has α elements. We know $X \backslash A_n$ is finite (and has say β elements) because $A_n \in \mathcal{T}$. But $X \backslash \{A_1 \cap A_2 \cap \ldots \cap A_{n-1} \cap A_n\} \subseteq X \backslash \{A_1 \cap A_2 \cap \ldots \cap A_{n-1}\} \cup X \backslash A_n$, so $X \backslash \{\bigcap_{k=1}^{n} A_n\}$ has at most $\alpha + \beta$ elements, which is finite.