

14 (3). *If $<$ is a simple order on X , then the associated order \leq is a total order on X .*

$<$ is a simple order so $<$ is an order.

By 14(2), \leq is a partial order.

We need to show that $x \leq y$ or $y \leq x$ for $x, y \in X$.

Let $x, y \in X$, and suppose that $x \not\leq y$. We want to show that $y \leq x$.

Then $x \not< y$ and $x \neq y$.

Since $<$ is a simple order, at least one of $x < y$, $x = y$ or $y < x$ must be true.

The first two aren't true, so $y < x$.

This means $y \leq x$, and we're done.