Jesse Gage

Math <Course>

Dr. <Prof>

October 31, 2006

Homework: That stupid #12 problem that I still can't do why cant i do math oh god

12. X, d and X, ρ have the same convergent sequences $\Rightarrow$ they have the same open sets

Let (X, d) and (X, ρ) have the same convergent sequences.

Suppose $A \subseteq X$ open in d

Need to show A_ρ

Since A_d open, A_d^c is closed

WTS A_ρ^c is closed; that is $x \in \overline{A_\rho^c} \Rightarrow x \in A_\rho^c$

Suppose $x \notin A_\rho^c$

then $x \notin A_d^c$

But A_d^c is closed, so $x \notin \overline{A_d^c}$

so there is no sequence in A_d^c that can converge to x

But (X, d) and (X, ρ) have the same convergent sequences

So there's no sequence in A_ρ^c that can converge ot x

Therefore $x \notin \overline{A_\rho^c}$

thus $x \notin A_\rho^c \Rightarrow x \notin \overline{A_\rho^c}$

$x \in \overline{A_\rho^c} \Rightarrow x \in A_\rho^c$ by contraposition

So A_ρ^c is closed

so A_ρ is open. $\square$