Show by example that if $f_n \to f$ uniformly $g_n \to g$ uniformly $\not\Rightarrow f_n g_n \to fg$ uniformly on $[a,b]$.
Let $f_n(x) = 1 + \frac{1}{n}$ and

$$g_n(x) = \begin{cases} q + \frac{1}{n}, & x = \frac{p}{q}, (p,q) = 1 \\ \frac{1}{n}, & x \notin \mathbb{Q} \end{cases}$$

on the interval $[1,2]$. Clearly, f_n is uniformly convergent to 1. The case for g_n converging to

$$g(x) = \begin{cases} q, & x = \frac{p}{q}, (p,q) = 1 \\ 0, & x \notin \mathbb{Q} \end{cases}$$

is somewhat less obvious until we note that $|g_n(x) - g(x)| = \frac{1}{n}$ for all $x \in [1,2]$, which can be made less than $\varepsilon > 0$. However,

$$|f_n g_n - fg| = \left| \begin{cases} \frac{1}{n} + \frac{q}{n} + \frac{1}{n^2} & x = \frac{p}{q}, (p,q) = 1 \\ \frac{1}{n} & x \notin \mathbb{Q} \end{cases} \right|$$

cannot be made less than an arbitrary $\varepsilon > 0$; indeed, if Q is some integer larger than ε then for any $n \in \mathbb{N}$ we have $\frac{nQ+1}{nQ} \in [1,2]$ and

$$\left| [f_n g_n - fg] \left(\frac{nQ+1}{nQ} \right) \right| = Q + \frac{1}{n} + \frac{1}{n^2} > Q > \varepsilon.$$